# Generalized Net Model of Overall Telecommunication System with Queuing

Velin Andonov, Stoyan Poryazov and Emiliya Saranova

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Block 8, 1113 Sofia, Bulgaria,
velin_andonov@math.bas.bg, stoyan@math.bas.bg, emiliya@math.bas.bg

**Abstract.** Generalized Net (GN) model of overall telecommunication system with queuing is proposed. It is based on the classical conceptual model of overall telecommunication system which considers user's behaviour, finite number of users and terminals, losses due to abandoned and interrupted dialing, blocked and interrupted switching, unavailable intent terminal, blocked and abandoned ringing and abandoned communication. A queuing system with finite capacities of the server and buffer and FIFO discipline of service of the requests is included in the Switching stage. The proposed model can be used in the analytical modeling of overall telecommunication systems.

**Keywords:** Generalized nets, Overall telecommunication system · Conceptual modeling.

## 1 Introduction

The classical conceptual model of overall telecommunication system considers user's behaviour, finite number of users and terminals, losses due to abandoned and interrupted dialing, blocked and interrupted switching, unavailable intent terminal, blocked and abandoned ringing and abandoned communication. The traffic of the calling (denoted by A) and the called (denoted by B) terminals and user's traffic are considered separately but in their interrelation. It is described in [7] and developed in more details in [8]. The model uses the concepts of Service Systems Theory.

Generalized Nets (GNs, see [3]) have been used as an alternative approach to the conceptual modeling of telecommunication systems starting with the paper [6]. More recently, a GN model of the Switching stage of overall telecommunication network is proposed in [9] and it is compared to a model based on Service Systems Theory. The research on the GNs as a tool for conceptual modeling of service systems continues in [10, 12] where GN models of queuing systems are proposed. In [1], for the first time, a queuing system is included in a GN model of a part of overall telecommunication system.

The first goal of the present paper is to construct a GN model of overall telecommunication system with queuing in the Switching stage. The second goal is to demonstrate how the GN model can be used to derive some analytical expressions for the parameters of the system.

In Section 2, the classical conceptual model of overall telecommunication system which is the base of the new GN model is presented briefly. The GN model of the overall telecommunication system with queuing is divided formally into four parts: GN model of the Dialing stage, GN model of the Switching stage, GN model of the Ringing stage and GN model of the Communication stage. They are described in Subsection 3.1 - 3.4, respectively. In Section 4, classification of the parameters of the system is proposed and assumptions about the system are formulated. Analytical expression for the traffic intensity of the called terminals is derived.

## 2 Classical conceptual model of overall telecommunication system

The classical model of overall telecommunication system is proposed in [7] and developed in more details in [8]. It is a detailed conceptual traffic model of an overall (virtual) circuit switching telecommunication network, like PSTN and GSM, including users' behaviour, with: BPP (Bernoulli-Poisson-Pascal) input flow; repeated calls; limited number of homogeneous terminals; losses due to abandoned and interrupted dialing, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing and abandoned communication.

The described approach is applicable directly to every (virtual) circuit switching telecommunication system (like GSM and PSTN) and may help considerably for ISDN, BISDN and most of core and access networks traffic modelling. For packet switching systems, like Internet, the proposed approach may be used as a comparison basis.

The traffic of the calling (denoted by A) and the called (denoted by B) terminals and user's traffic are considered separately but in their interrelation. Two types of virtual devices are included in the model: base and comprise devices.

### 2.1 Base virtual devices representation and their parameters

At the bottom of the structural model presentation, we consider base virtual devices that do not contain any other virtual devices. A base virtual device has a general graphical representation as shown in Fig. 1.

The parameters of the base virtual device x are the following (see [5] for terms definition):

- $Fx$ - intensity or incoming rate (frequency) of the flow of requests (i.e. the number of requests per time unit) to device x;
- $Px$ - probability of directing the requests towards device x;
- $Tx$ - service time (duration of service of a request) in device x;
- $Yx$ - traffic intensity [Erlang];

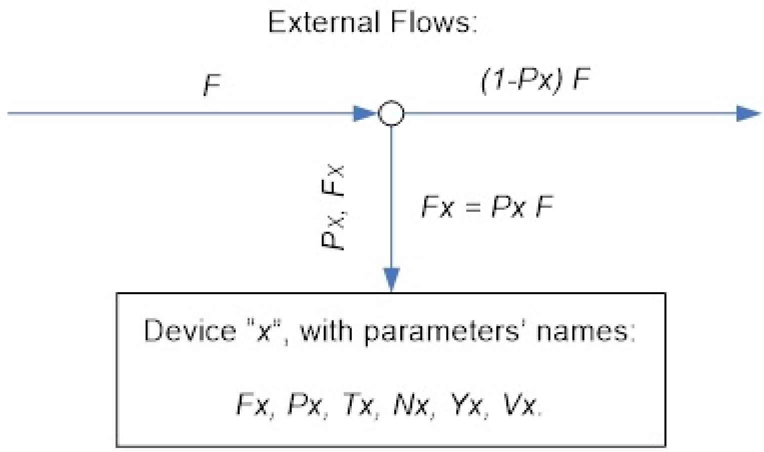


**Fig. 1.** Graphical representation of a base virtual device x.

- $Vx$ - traffic volume [Erlang - time unit];
- $Nx$ - number of lines (service resources, positions, capacity) of device x.

For the better understanding of the model and for a more convenient description of the intensity of the flow, a special notation including qualifiers (see [5]) is used. For example $dem.F$ stands for demand flow; $inc.Y$ stands for incoming traffic; $ofr.Y$ for offered traffic; $rep.Y$ for repeated traffic.

### 2.2 Types and names of the base virtual devices

The graphic representations of the base virtual devices together with their names and types are shown in Fig. 2 (see [7]). Each base virtual device belongs to one of the following types: Generator, Terminator, Modifier, Server, Enter Switch, Switch and Graphic connector. With the exception of the Switch, which has one or two entrances and one or two exits, every other virtual device has one entrance and/or one exit.

In the conceptual model, each virtual device has a unique name. The names of the devices are constructed according to their position in the model. The model is partitioned into service stages (dialing, switching, ringing and communication). Every service stage has branches (enter, abandoned, blocked, interrupted, not available, carried), corresponding to the modeled possible cases of ends of the calls' service in the branch considered. Every branch has two exits (repeated, terminated) which show what happens with the calls if they can not be serviced by some device. Users may make a new bid (repeated call), or stop the attempts (terminated call). In the virtual device name construction, the corresponding bold first letters of the names of stages, branches end exits are used in the order shown below.

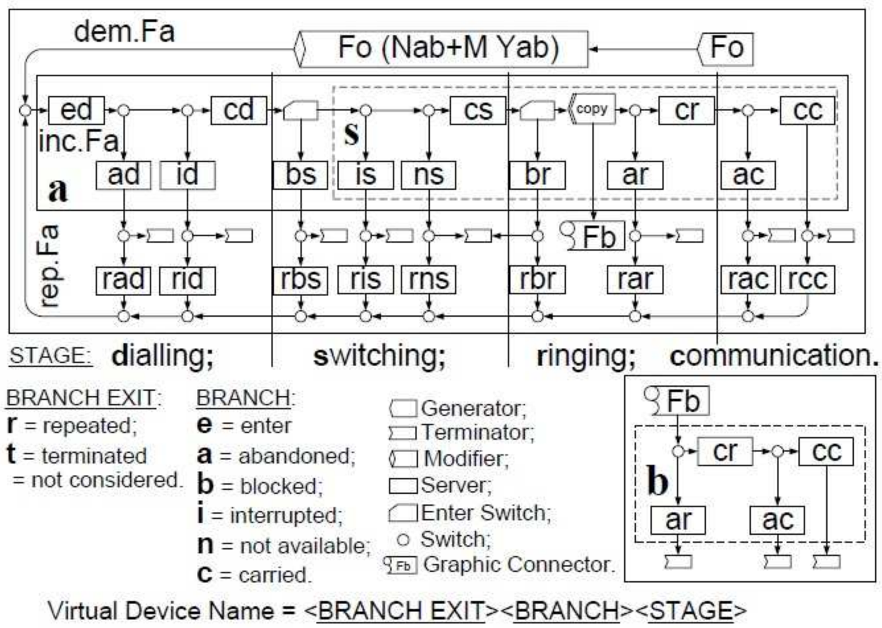


**Fig. 2.** Classical conceptual model of an overall telecommunication system (see [7]).

Virtual Device Name = <BRANCH EXIT><BRANCH><STAGE>

The parameter's name of a virtual device is a concatenation of parameter's name letter and virtual device name. For example, "Yid" means "traffic intensity in interrupted dialing case"; "Fid" means "flow (calls) intensity in interrupted dialing case"; "Pid" means "probability for interrupted dialing"; Tid = "mean duration of the interrupted dialing"; "Frid" = "intensity of repeated flow calls, caused by (after) interrupted dialing".

### 2.3 Comprise virtual devices

The following comprise virtual devices denoted by **a, b, s** (see Fig. 2) and **ab** (not shown in Fig. 2) are considered in the model.

- **a** comprises all calling terminals (A-terminals) in the system. It is shown with continuous line box in Fig. 2;
- **b** comprises all called terminals (B-terminals) in the system. It is shown in box with dashed line in the down right corner in Fig. 2;
- **ab** comprises all the terminals (calling and called) in the system. It is not shown in Fig. 2;
- **s** virtual device corresponding to the switching system. It is shown with dashed line box into the **a**-device in Fig. 2.

# 3 Generalized net model of overall telecommunication system with queuing

Using the classical model of overall telecommunication system described in the previous section and the GN representations of the basic elements of Service Systems Theory proposed in [2], we shall construct a GN model of overall telecommunication system with queuing. The buffer of the queuing system we denote by **ws** (from "waiting for switch"). The queuing system considered can be represented in Kendall's notation (see [4, 11]) as $M|M|Ns|Ns + Nws|Nab|FIFO$, where $M$ stands for exponential distribution, $Ns$ is the capacity of the Switching system (number of equivalent internal switching lines), $Nws$ is the capacity of the buffer device and $Nab$ is the total number of active terminals which can be calling and called. This is related to the analytical modeling of the system.

The GN consists of 29 transitions and 77 places. Among these 77 places, 28 correspond to base virtual devices of the classical model. Their labels are in the form $l_x$ where "x" is a name of a base virtual device (see Fig. 2). Due to the huge number of transitions and places, the GN model is formally divided into 4 parts corresponding to the four stages of the classical model: Dialing, Switching, Ringing and Communication.

## 3.1 Generalized net model of the Dialing stage

The GN model of the Dialing is shown in Fig. 3. It consists of 7 transitions and 25 places. The transitions represent functions of base virtual devices as follows:

- $Z_1$ represents the function of the Modifier from Fig. 2;
- $Z_2$ represents the **cd** device;
- $Z_3$ represents the function of the **ad** device;
- $Z_4$ represents the function of the **rad** device;
- $Z_5$ represents the function of the **id** device;
- $Z_6$ represents the function of the **rid** device;
- $Z_7$ represents the function of the **cd** device.

Among the 25 places, 7 are not shown in Fig. 3. They are output places of transitions from the other three stages. The places corresponding to base virtual devices are $l_{Fo}, l_{ed}, l_{ad}, l_{rad}, l_{id}, l_{rid}, l_{cd}$. In place $l_{Fo}$ token of type $\alpha$ stays in the initial time moment with characteristic *"formula for generating the demand flow of requests"*. Tokens $\alpha_{ed}, \alpha_{ad}, \alpha_{rad}, \alpha_{id}, \alpha_{rid}, \alpha_{cd}$ stay in the initial time moment in places $l_{ed}, l_{ad}, l_{rad}, l_{id}, l_{rid}, l_{cd}$, respectively. Each of them has initial characteristic *"initial values of* $Y_x, P_x, F_x, T_x, N_x$*"*, where $x$ is the name of the corresponding base virtul device.

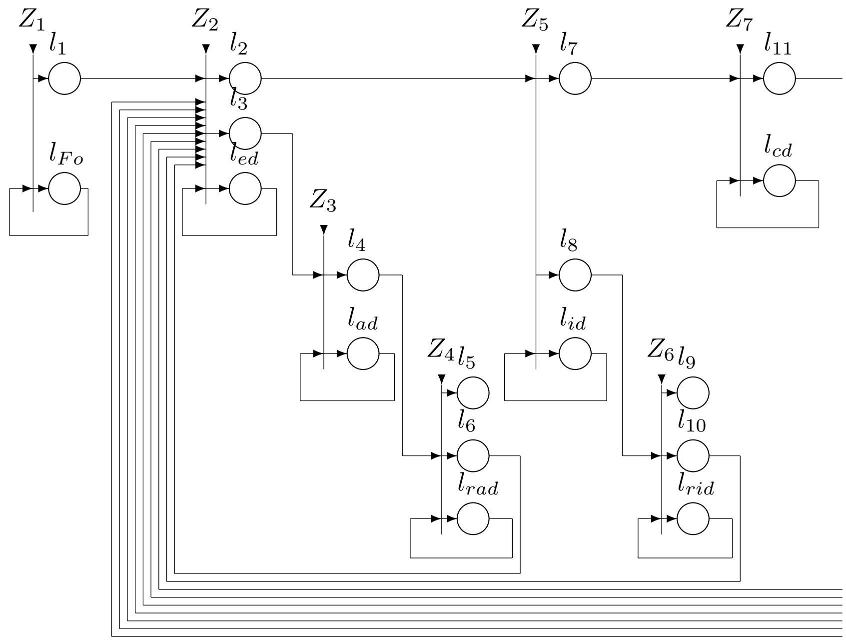


Fig. 3. Generalized net model of the Dialiang stage.

Below is a formal description of the transitions.

$$Z_1 = \langle \{l_{Fo}\}, \{l_1, l_{Fo}\}, r_1 \rangle,$$

where

$$r_1 = \begin{array}{c|cc} & l_1 & l_{Fo} \\ \hline l_{Fo} & W_{Fo,1} & true \end{array}$$

and

- $W_{Fo,1}$ = "A demand flow of requests is generated".

When the truth value of the predicate $W_{Fo,1}$ is "true", the $\alpha$ token splits into two tokens – the same $\alpha$ token which stays in place $l_{Fo}$ and $\alpha_1$ which enters place $l_1$ with characteristic *"intensity of the demand flow of requests"*. The token $\alpha$ in place $l_{Fo}$ obtains the characteristic *"current formula for generation of the demand flow of requests"*.

$$Z_2 = \langle \{l_1, l_6, l_{10}, l_{15}, l_{19}, l_{23}, l_{28}, l_{33}, l_{44}, l_{47}, l_{ed}\}, \{l_2, l_3, l_{ed}\}, r_2 \rangle,$$

where

$$r_2 = \begin{array}{c|ccc} & l_2 & l_3 & l_{ed} \\ \hline l_1 & false & false & true \\ l_6 & false & false & true \\ l_{10} & false & false & true \\ l_{15} & false & false & true \\ l_{19} & false & false & true \\ l_{23} & false & false & true \\ l_{28} & false & false & true \\ l_{33} & false & false & true \\ l_{44} & false & false & true \\ l_{47} & false & false & true \\ l_{ed} & W_{ed,2} & W_{ed,3} & true \end{array}$$

and

- $W_{ed,2}$ =“the current request is not abandoned (with a given probability)”;
- $W_{ed,3}$ = “the current request is abandoned (with a given probability)”.

The tokens from all input places of the transition $Z_2$ merge with the token $\alpha_{ed}$ in place $l_{ed}$. Depending on the truth values of the predicates $W_{ed,2}$ and $W_{ed,3}$ token $\alpha_{ed}$ may split into two or three tokens. If both predicates have truth value “true”, it splits into three tokens – the same $\alpha_{ed}$, $\alpha_1$ and $\alpha_2$. Token $\alpha_{ed}$ obtains the characteristic *“current value of Yed”*. Tokens $\alpha_1$ and $\alpha_2$ enter places $l_2$ and $l_3$ respectively without obtaining new characteristic.

$$Z_3 = \langle \{l_3, l_{ad}\}, \{l_4, l_{ad}\}, r_3 \rangle,$$

where

$$r_3 = \begin{array}{c|cc} & l_4 & l_{ad} \\ \hline l_3 & false & true \\ l_{ad} & true & true \end{array}.$$

The $\alpha_2$ token from place $l_3$ merges with the $\alpha_{ad}$ token in place $l_{ad}$. The $\alpha_{ad}$ token splits into two tokens – the same $\alpha_{ad}$ token which stays in playce $l_{ad}$ with characteristic *“current value of Yad”* and token $\alpha_3$ which enters place $l_4$ without obtaining new characteristic.

$$Z_4 = \langle \{l_4, l_{rad}\}, \{l_5, l_6, l_{rad}\}, r_4 \rangle,$$

where

$$r_4 = \begin{array}{c|ccc} & l_5 & l_6 & l_{rad} \\ \hline l_4 & W_{4,5} & false & W_{4,rad} \\ l_{rad} & false & true & true \end{array},$$

and

- $W_{4,5}$ = “the current call is terminated (with a given probability)”;
- $W_{4,rad}$ = “the current call is repeated (with a given probability)”.

When the truth value of predicate $W_{4,5}$ is "true" token $\alpha_3$ enters place $l_5$ without obtaining new characteristic. When the truth value of $W_{4,rad}$ is "true" token $\alpha_3$ enters place $l_{rad}$ where it merges with token $\alpha_{rad}$. Token $\alpha_{rad}$ splits into two tokens – the same $\alpha_{rad}$ which remains in place $l_{rad}$ with characteristic *"current value of Yrad"* and token $\alpha_4$ which enters place $l_6$ without obtaining new characteristic.

$$Z_5 = \langle \{l_2, l_{id}\}, \{l_7, l_8, l_{id}\}, r_5 \rangle,$$

where

$$r_5 = \begin{array}{c|ccc} & l_7 & l_8 & l_{id} \\ \hline l_2 & W_{2,7} & false & W_{2,id} \\ l_{id} & false & true & true \end{array},$$

and

- $W_{2,7}$ = "the current call is carried (with a given probability)";
- $W_{2,id}$ = "the current call is interrupted (with a given probability)".

When the truth value of predicate $W_{2,7}$ is "true" token $\alpha_1$ enters place $l_7$ without obtaining new characteristic. When the truth value of $W_{2,id}$ is "true" token $\alpha_1$ enters place $l_{id}$ where it merges with token $\alpha_{id}$. Token $\alpha_{id}$ splits into two tokens – the same $\alpha_{id}$ which remains in place $l_{id}$ with characteristic *"current value of Yid"* and token $\alpha_5$ which enters place $l_8$ without obtaining new characteristic.

$$Z_6 = \langle \{l_8, l_{rid}\}, \{l_9, l_{10}, l_{rid}\}, r_6 \rangle,$$

where

$$r_6 = \begin{array}{c|ccc} & l_9 & l_{10} & l_{rid} \\ \hline l_8 & W_{8,9} & false & W_{8,rid} \\ l_{rid} & false & true & true \end{array},$$

and

- $W_{8,9}$ = "the current call is terminated (with a given probability)";
- $W_{8,rid}$ ="the current call is repeated (with a given probability)".

When the truth value of predicate $W_{8,9}$ is "true" token $\alpha_5$ enters place $l_9$ without obtaining new characteristic. When the truth value of $W_{8,rid}$ is "true" token $\alpha_5$ enters place $l_{rid}$ where it merges with token $\alpha_{rid}$. Token $\alpha_{rid}$ splits into two tokens – the same $\alpha_{rid}$ which remains in place $l_{rid}$ with characteristic *"current value of Yrid"* and token $\alpha_6$ which enters place $l_{10}$ without obtaining new characteristic.

$$Z_7 = \langle \{l_7, l_{cd}\}, \{l_{11}, l_{cd}\}, r_7 \rangle,$$

where

$$r_7 = \begin{array}{c|cc} & l_{11} & l_{cd} \\ \hline l_7 & true & true \\ l_{cd} & false & true \end{array}.$$

Token $\alpha_1$ from $l_7$ splits into two tokens one of which merges with token $\alpha_{cd}$ in place $l_{cd}$ and the other one enters place $l_{11}$ without obtaining new characteristic. Token $\alpha_{cd}$ obtains the characteristic *"current value of Ycd"*.

### 3.2 Generalized net model of the Switching stage

The graphical representation of the GN model of the Switching stage with queuing is shown in Fig. 4.

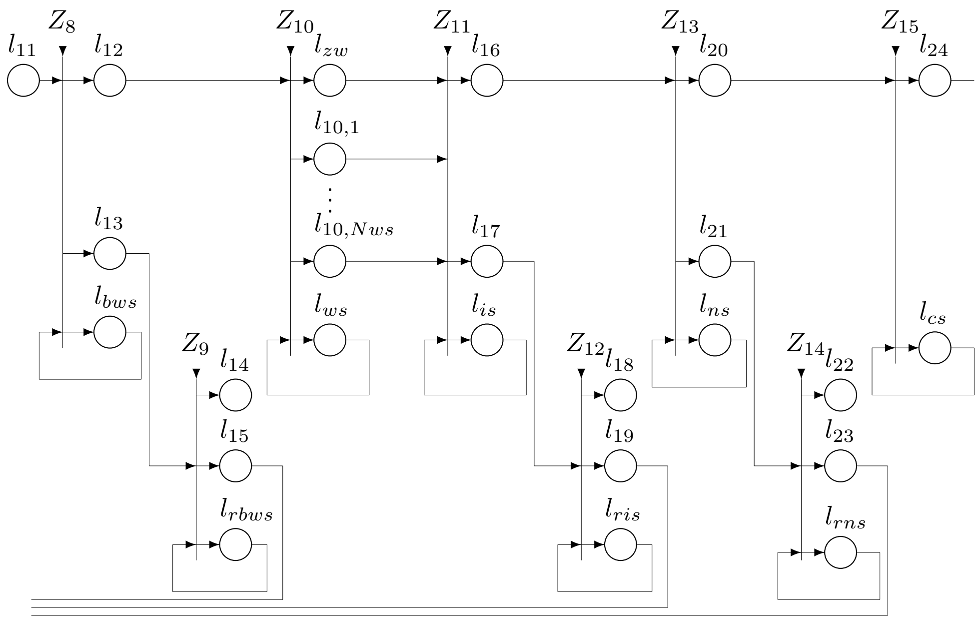


Fig. 4. Generalized net model of the Switching stage of an overall telecommunication system with queuing.

It consists of 8 transitions and $23 + Nws$ places where $Nws$ is the capacity of the buffer of the queuing system. The transitions represent functions of base virtual devices as follows:

- $Z_8$ represents the function of the Enter Switch device before the comprise virtual device **s** in Fig. 2;
- $Z_9$ represents the function of Switch device after the **bws** device. The **bws** is a new device analogous to the **bs** device in Fig. 2;
- $Z_{10}$ represents the function of the buffer device;
- $Z_{11}$ represents the function of the Switch device before the **is** device;
- $Z_{12}$ represents the function of the Switch device after the **is** device;
- $Z_{13}$ represents the function of the Switch device before the **cs** device;
- $Z_{14}$ represents the function of the Switch device after the **ns** device.
- $Z_{15}$ represents the function of the **cs** device.

Among the $23 + Nws$ places, 8 places correspond to base virtual devices. These are the places with labels $l_{bws}, l_{rbws}, l_{ws}, l_{is}, l_{ris}, l_{ns}, l_{rns}, l_{cs}$. Tokens $\alpha_{bws}$, $\alpha_{rbws}$, $\alpha_{ws}, \alpha_{is}, \alpha_{ris}, \alpha_{ns}, \alpha_{rns}, \alpha_{cs}$ stay in the initial time moment in places $l_{bws}$, $l_{rbws}$,

$l_{ws}, l_{is}, l_{ris}, l_{ns}, l_{rns}, l_{cs}$, respectively. Each of them has initial characteristic *"initial values of* $Y_x, P_x, F_x, T_x, N_x$*"*, where $x$ is the name of the corresponding base virtul device.

The queuing system in the Switching stage is represented in the GN by transitions $Z_{10}$ and $Z_{11}$. There are other possible representations of queuing systems by GNs (see [10, 12]). The one presented here shows explicitly the two ways of servicing of the requests by the queuing system – with waiting and without waiting. When the Switching system has not reached its capacity the tokens enter the place $l_{zw}$ (zero waiting) which corresponds to the service without waiting. If the Switching system has reached its capacity, the tokens enter some of the places $l_{10,1}, ..., l_{10,Nws}$ which corresponds to the waiting in the buffer.

Below is a formal description of the transitions of the GN model of the switching stage.

$$Z_8 = \langle \{l_{11}, l_{bws}\}, \{l_{12}, l_{13}, l_{bws}\}, r_8 \rangle,$$

where

$$r_8 = \begin{array}{c|ccc} & l_{12} & l_{13} & l_{bws} \\ \hline l_{11} & W_{11,12} & false & W_{11,bws} \\ l_{bws} & false & true & true \end{array},$$

and

- $W_{11,12} =$ "$Yws < Nws$";
- $W_{11,bws} = \neg W_{11,12}$.

When the truth value of predicate $W_{11,12}$ is "true" token $\alpha_1$ enters place $l_{12}$ without obtaining new characteristic. When the truth value of $W_{11,bws}$ is "true" token $\alpha_1$ enters place $l_{bws}$ where it merges with token $\alpha_{bws}$. Token $\alpha_{bws}$ splits into two tokens – the same $\alpha_{bws}$ which remains in place $l_{bws}$ with characteristic *"current value of* $Ybws$*"* and token $\alpha_7$ which enters place $l_{13}$ without obtaining new characteristic.

$$Z_9 = \langle \{l_{13}, l_{rbws}\}, \{l_{14}, l_{15}, l_{rbws}\}, r_9 \rangle,$$

where

$$r_9 = \begin{array}{c|ccc} & l_{14} & l_{15} & l_{rbws} \\ \hline l_{13} & W_{13,14} & false & W_{13,rbws} \\ l_{rbws} & false & true & true \end{array},$$

and

- $W_{13,14} =$ "the current request is terminated (with given probability)";
- $W_{13,rbws} =$ "the current request becomes repeated request (with given probability)".

When the truth value of predicate $W_{13,14}$ is "true" token $\alpha_7$ enters place $l_{14}$ without obtaining new characteristic. When the truth value of $W_{13,rbws}$ is "true" token $\alpha_7$ enters place $l_{rbws}$ where it merges with token $\alpha_{rbws}$. Token $\alpha_{rbws}$ splits into two tokens – the same $\alpha_{rbws}$ which remains in place $l_{rbws}$ with characteristic *"current value of Yrbws"* and token $\alpha_8$ which enters place $l_{15}$ without obtaining new characteristic.

$$Z_{10} = \langle \{l_{12}, l_{ws}\}, \{l_{zw}, l_{10,1}, ..., l_{10,Nws}, l_{ws}\}, r_{10} \rangle,$$

where

$$r_{10} = \begin{array}{c|ccccc} & l_{zw} & l_{10,1} & \ldots & l_{10,Nws} & l_{ws} \\ \hline l_{12} & W_{12,zw} & W_{12,101} & \ldots & W_{12,10Nws} & W_{12,ws} \\ l_{ws} & false & false & \ldots & false & true \end{array},$$

and

- $W_{12,zw} =$ "$Ys < Ns$";
- $W_{12,10i} = \neg W_{12,zw}$ & "place $l_{10,i}$ is the highest priority empty place among places $l_{10,1}, ..., l_{10,Nws}$" for $i = 1, 2, ..., Nws$;
- $W_{12,ws} = W_{12,101} \vee W_{12,102} \vee .... \vee W_{12,10Nws}$.

When the truth value of predicate $W_{12,zw}$ is "true" token $\alpha_1$ enters place $l_{zw}$ without obtaining new characteristic. When the truth value of the predicate $W_{12,10i}$ for $i = 1, 2, ..., Nws$ is "true" token $\alpha_1$ splits into as many tokens as the number of the predicates with truth value "true" which enter the corresponding buffer places without new characteristic, and a token $\alpha_9$ which enter place $l_{ws}$ where it merges with token $\alpha_{ws}$. Token $\alpha_{ws}$ splits into two tokens – the same $\alpha_{ws}$ which remains in place $l_{ws}$ with characteristic *"current value of Yws; list of all tokens in places $l_{10,1}, ..., l_{10,Nws}$ and the duration of tokens stay in the places"*.

$$Z_{11} = \langle \{l_{zw}, l_{10,1}, ..., l_{10,Nws}, l_{is}\}, \{l_{16}, l_{17}, l_{is}\}, r_{11} \rangle,$$

where

$$r_{11} = \begin{array}{c|ccc} & l_{16} & l_{17} & l_{is} \\ \hline l_{zw} & W_{zw,16} & false & W_{zw,is} \\ l_{10,1} & W_{101,16} & false & W_{101,is} \\ \vdots & \vdots & \vdots & \vdots \\ l_{10,Nws} & W_{10Nws,16} & false & W_{10Nws,is} \\ l_{is} & false & true & true \end{array},$$

and

- $W_{zw,is}$ ="the current request is interrupted (with a given probability)";
- $W_{zw,16} = \neg W_{zw,is}$;
- $W_{10i,is}$ ="the current request is interrupted (with a given probability)" & "the token in place $l_{10i,is}$ has stayed in the place more time compared to the tokens in the rest of the input places $l_{10,1}, l_{10,2}, ..., l_{10,Nws}$" for $i = 1, 2, ..., Nws$;
- $W_{10i,16}$ = "the current request is not interrupted (with a given probability)" & "the token in place $l_{10,i}$ has stayed in the place more time compared to the tokens in the rest of the input places $l_{10,1}, l_{10,2}, ..., l_{10,Nws}$" for $i = 1, 2, ..., Nws$;

When the truth value of predicate $W_{zw,16}$ is "true" token $\alpha_1$ from place $l_{zw}$ enters place $l_{16}$ without obtaining new characteristic. When the truth value of some of the predicates $W_{101,16}, W_{102,16}, ..., W_{10Nws,16}$ is "true" the corresponding $\alpha_1$ tokens enter place $l_{16}$ without obtaining new characteristic. When the truth value of the predicate $W_{zw,is}$ is "true" token $\alpha_1$ from place $l_{zw}$ enters place $l_{is}$ where it merges with the $\alpha_{is}$ token. When the truth value of some of the predicates $W_{10i,is}$ for $i = 1, 2, ..., Nws$ is "true" the corresponding token $\alpha_1$ enters place $l_{is}$ where it merges with the $\alpha_{is}$ token. Token $\alpha_{is}$ splits into two tokens – the same $\alpha_{is}$ which remains in place $l_{is}$ with characteristic *"current value of Yis"* and token $\alpha_9$ which enters place $l_{17}$ without obtaining characteristic.

$$Z_{12} = \langle \{l_{17}, l_{ris}\}, \{l_{18}, l_{19}, l_{ris}\}, r_{12} \rangle,$$

where

$$r_{12} = \begin{array}{c|ccc} & l_{18} & l_{19} & l_{ris} \\ \hline l_{17} & W_{17,18} & false & W_{17,ris} \\ l_{ris} & false & true & true \end{array}$$

and

- $W_{17,18}$ = "the current request is terminated (with a given probability)";
- $W_{17,ris}$ = "the current request becomes repeated request (with a given probability)".

When the truth value of predicate $W_{17,18}$ is "true" token $\alpha_9$ from place $l_{17}$ enters place $l_{18}$ without obtaining new characteristic. When the truth value of $W_{17,ris}$ is "true" token $\alpha_9$ enters place $l_{ris}$ where it merges with token $\alpha_{ris}$. Token $\alpha_{ris}$ splits into two tokens – the same $\alpha_{ris}$ which remains in place $l_{ris}$ with characteristic *"current value of Yris"* and token $\alpha_{10}$ which enters place $l_{19}$ without obtaining new characteristic.

$$Z_{13} = \langle \{l_{16}, l_{ns}\}, \{l_{20}, l_{21}, l_{ns}\}, r_{13} \rangle,$$

where

$$r_{13} = \begin{array}{c|ccc} & l_{20} & l_{21} & l_{ns} \\ \hline l_{16} & W_{16,20} & false & W_{16,ns} \\ l_{ns} & false & true & true \end{array},$$

and

- $W_{16,ns}$ = "not available switching for the current request (with a given probability)";
- $W_{16,20} = \neg W_{16,ns}$.

When the truth value of predicate $W_{16,20}$ is "true" token $\alpha_1$ from place $l_{16}$ enters place $l_{20}$ without obtaining new characteristic. When the truth value of $W_{16,ns}$ is "true" token $\alpha_1$ enters place $l_{ns}$ where it merges with token $\alpha_{ns}$. Token $\alpha_{ns}$ splits into two tokens – the same $\alpha_{ns}$ which remains in place $l_{ns}$

with characteristic *"current value of Yns"* and token $\alpha_{11}$ which enters place $l_{21}$ without obtaining new characteristic.

$$Z_{14} = \langle\{l_{21}, l_{rns}\}, \{l_{22}, l_{23}, l_{rns}\}, r_{14}\rangle,$$

where

$$r_{14} = \begin{array}{c|ccc} & l_{22} & l_{23} & l_{rns} \\ \hline l_{21} & W_{21,22} & false & W_{21,rns} \\ l_{rns} & false & true & true \end{array}$$

and

- $W_{21,22}$ = "the current request is terminated (with a given probability)";
- $W_{21,rns}$ = "the current request becomes repeated request (with a given probability)".

When the truth value of predicate $W_{21,22}$ is "true" token $\alpha_{11}$ from place $l_{21}$ enters place $l_{22}$ without obtaining new characteristic. When the truth value of $W_{21,rns}$ is "true" token $\alpha_{11}$ enters place $l_{rns}$ where it merges with token $\alpha_{rns}$. Token $\alpha_{rns}$ splits into two tokens – the same $\alpha_{rns}$ which remains in place $l_{rns}$ with characteristic *"current value of Yrns"* and token $\alpha_{12}$ which enters place $l_{23}$ without obtaining new characteristic.

$$Z_{15} = \langle\{l_{20}, l_{cs}\}, \{l_{24}, l_{cs}\}, r_{15}\rangle,$$

where

$$r_{15} = \begin{array}{c|cc} & l_{24} & l_{cs} \\ \hline l_{20} & true & true \\ l_{cs} & false & true \end{array}.$$

Token $\alpha_1$ from place $l_{20}$ splits into two tokens – the same $\alpha_1$ which enters place $l_{24}$ without obtaining new characteristic and token $\alpha_{13}$ which enters place $l_{cs}$ where it merges with token $\alpha_{cs}$. Token $\alpha_{cs}$ obtains the characteristic *"current value of Ycs"*.

### 3.3 Generalized net model of the Ringing stage

The graphical representation of the GN model of the Ringing stage is shown in Fig. 5. It consists of 10 transitions and 27 places. The transitions represent functions of base virtual devices as follows:

- $Z_{16}$ represents the function of the Enter Switch device after the base virtual device **cs** in Fig. 2;
- $Z_{17}$ represents the function of Switch device after the **br** device in Fig. 2;
- $Z_{18}$ represents the function of the COPY device;
- $Z_{19}$ represents the function of the Switch device after the COPY device;
- $Z_{20}$ represents the function of the Switch device after the **ar** device;
- $Z_{21}$ represents the function of the **cr** device at the end of the Ringing stage in Fig. 2;

- $Z_{22}$ represents the function of the Switch device at the beginning of the comprise virtual device **b** of the called terminals in Fig. 2;
- $Z_{23}$ represents the function of the **cr** device in the comprise virtual device **b**;
- $Z_{24}$ represents the function of the Switch device after the **cr** device in the comprise virtual device **b**;
- $Z_{25}$ represents the function of the **cc** device in the comprise virtual device **b**.

Among the 27 places, 9 places correspond to base virtual devices. These are the places with labels $l_{br}, l_{rbr}, l_{ar}, l_{rar}, l_{cr}, l'_{ar}, l'_{cr}, l'_{ac}, l'_{cc}$. Tokens $\alpha_{br}, \alpha_{rbr}, \alpha_{ar}, \alpha_{rar}, \alpha_{cr}, \alpha'_{ar}, \alpha'_{cr}, \alpha'_{ac}, \alpha'_{cc}$ stay in the initial time moment in places $l_{br}, l_{rbr}, l_{ar}, l_{rar}, l_{cr}, l'_{ar}, l'_{cr}, l'_{ac}, l'_{cc}$, respectively. Each of them has initial characteristic *"initial values of* $Y_x, P_x, F_x, T_x, N_x$*"*, where $x$ is the name of the corresponding base virtual device.

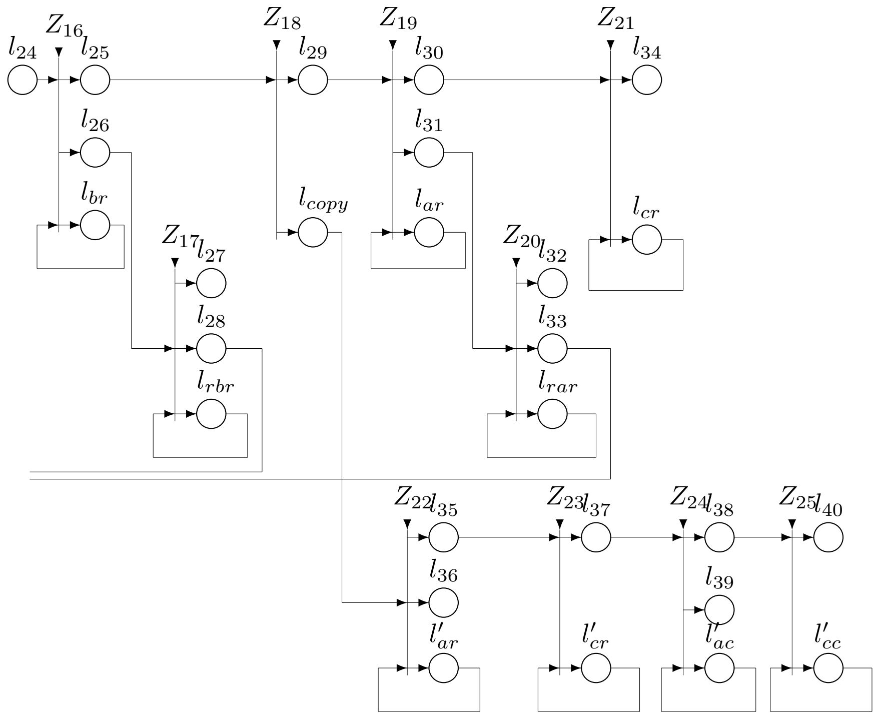


Fig. 5. Generalized net model of the Ringing stage of an overall telecommunication network with queuing.

Transitions $Z_{22}, Z_{23}, Z_{24}$ and $Z_{25}$ represent the comprise virtual device **b** of the called terminals. Although $Z_{24}$ and $Z_{25}$ are part of the Communication stage, they are included in the Ringing stage to avoid unnecessary complication of the graphical representation of the GN model.

Below is the formal description of the transitions.

$$Z_{16} = \langle \{l_{24}, l_{br}\}, \{l_{25}, l_{26}, l_{br}\}, r_{16} \rangle,$$

where

$$r_{16} = \begin{array}{c|ccc} & l_{25} & l_{26} & l_{br} \\ \hline l_{24} & W_{24,25} & false & W_{24,br} \\ l_{br} & false & true & true \end{array}$$

and

- $W_{24,br}$ = "the current request is blocked (with a given probability)";
- $W_{24,25} = \neg W_{24,br}$.

When the truth value of predicate $W_{24,25}$ is "true" token $\alpha_1$ from place $l_{24}$ enters place $l_{25}$ without obtaining new characteristic. When the truth value of $W_{24,br}$ is "true" token $\alpha_1$ enters place $l_{br}$ where it merges with token $\alpha_{br}$. Token $\alpha_{br}$ splits into two tokens – the same $\alpha_{br}$ which remains in place $l_{br}$ with characteristic *"current value of Ybr"* and token $\alpha_{14}$ which enters place $l_{26}$ without obtaining new characteristic.

$$Z_{17} = \langle \{l_{26}, l_{rbr}\}, \{l_{27}, l_{28}, l_{rbr}\}, r_{17} \rangle,$$

where

$$r_{17} = \begin{array}{c|ccc} & l_{27} & l_{28} & l_{rbr} \\ \hline l_{26} & W_{26,27} & false & W_{26,rbr} \\ l_{rbr} & false & true & true \end{array}$$

and

- $W_{26,rbr}$ = "the current request is repeated (with given probability)";
- $W_{26,27}$ = "the current request is terminated (with given probability)".

When the truth value of predicate $W_{26,27}$ is "true" token $\alpha_{14}$ from place $l_{26}$ enters place $l_{27}$ without obtaining new characteristic. When the truth value of $W_{26,rbr}$ is "true" token $\alpha_{14}$ enters place $l_{rbr}$ where it merges with token $\alpha_{rbr}$. Token $\alpha_{rbr}$ splits into two tokens – the same $\alpha_{rbr}$ which remains in place $l_{rbr}$ with characteristic *"current value of Yrbr"* and token $\alpha_{15}$ which enters place $l_{28}$ without obtaining new characteristic.

$$Z_{18} = \langle \{l_{25}\}, \{l_{29}, l_{copy}\}, r_{18} \rangle,$$

where

$$r_{18} = \begin{array}{c|cc} & l_{29} & l_{copy} \\ \hline l_{25} & true & true \end{array}.$$

Token $\alpha_1$ from place $l_{25}$ splits into two identical tokens – the same token $\alpha_1$ and token $\alpha_{16}$ – which enter places $l_{29}$ and $l_{copy}$, respectively, without obtaining new characteristics.

$$Z_{19} = \langle \{l_{29}, l_{ar}\}, \{l_{30}, l_{31}, l_{ar}\}, r_{19} \rangle,$$

where

$$r_{19} = \begin{array}{c|ccc} & l_{30} & l_{31} & l_{ar} \\ \hline l_{29} & W_{29,30} & false & W_{29,ar} \\ l_{ar} & false & true & true \end{array}$$

and

- $W_{29,ar} =$ "the current request is abandoned (with given probability)";
- $W_{29,30} = \neg W_{29,ar}$.

When the truth value of predicate $W_{29,30}$ is "true" token $\alpha_1$ from place $l_{29}$ enters place $l_{30}$ without obtaining new characteristic. When the truth value of $W_{29,ar}$ is "true" token $\alpha_1$ enters place $l_{ar}$ where it merges with token $\alpha_{ar}$. Token $\alpha_{ar}$ splits into two tokens – the same $\alpha_{ar}$ which remains in place $l_{ar}$ with characteristic *"current value of Yar"* and token $\alpha_{17}$ which enters place $l_{31}$ without obtaining new characteristic.

$$Z_{20} = \langle \{l_{31}, l_{rar}\}, \{l_{32}, l_{33}, l_{rar}\}, r_{20} \rangle,$$

where

$$r_{20} = \begin{array}{c|ccc} & l_{32} & l_{33} & l_{rar} \\ \hline l_{31} & W_{31,32} & false & W_{31,rar} \\ l_{rar} & false & true & true \end{array}$$

and

- $W_{31,rar} =$ "the current request is repeated (with given probability)";
- $W_{31,32} =$ "the current request is terminated (with given probability)".

When the truth value of predicate $W_{31,32}$ is "true" token $\alpha_{17}$ from place $l_{31}$ enters place $l_{32}$ without obtaining new characteristic. When the truth value of $W_{31,rar}$ is "true" token $\alpha_{17}$ enters place $l_{rar}$ where it merges with token $\alpha_{rar}$. Token $\alpha_{rar}$ splits into two tokens – the same $\alpha_{rar}$ which remains in place $l_{rar}$ with characteristic *"current value of Yrar"* and token $\alpha_{18}$ which enters place $l_{33}$ without obtaining new characteristic.

$$Z_{21} = \langle \{l_{30}, l_{cr}\}, \{l_{34}, l_{cr}\}, r_{21} \rangle,$$

where

$$r_{21} = \begin{array}{c|cc} & l_{34} & l_{cr} \\ \hline l_{30} & true & true \\ l_{cr} & false & true \end{array}.$$

Token $\alpha_1$ from place $l_{30}$ splits into two tokens – the same $\alpha_1$ which enters place $l_{34}$ without new characteristic and token $\alpha_{19}$ which enters place $l_{cr}$ where it merges with token $\alpha_{cr}$. Token $\alpha_{cr}$ obtains the characteristic *"current value of Ycr"*.

$$Z_{22} = \langle \{l_{copy}, l'_{ar}\}, \{l_{35}, l_{36}, l'_{ar}\}, r_{22} \rangle,$$

where

$$r_{22} = \begin{array}{c|ccc} & l_{35} & l_{36} & l'_{ar} \\ \hline l_{copy} & W_{copy,35} & false & W_{copy,ar} \\ l'_{ar} & false & true & true \end{array},$$

and

- $W_{copy,ar}$ = "the current request is abandoned (with given probability)";
- $W_{copy,35} = \neg W_{copy,ar}$.

When the truth value of predicate $W_{copy,35}$ is "true" token $\alpha_{16}$ from place $l_{copy}$ enters place $l_{35}$ without obtaining new characteristic. When the truth value of $W_{copy,ar}$ is "true" token $\alpha_{16}$ enters place $l'_{ar}$ where it merges with token $\alpha'_{ar}$. Token $\alpha'_{ar}$ splits into two tokens – the same $\alpha'_{ar}$ which remains in place $l'_{ar}$ with characteristic *"current value of Yar"* and token $\alpha_{20}$ which enters place $l_{36}$ without obtaining new characteristic.

$$Z_{23} = \langle \{l_{35}, l'_{cr}\}, \{l_{37}, l'_{cr}\}, r_{23} \rangle,$$

where

$$r_{23} = \begin{array}{c|cc} & l_{37} & l'_{cr} \\ \hline l_{35} & true & true \\ l'_{cr} & false & true \end{array}.$$

Token $\alpha_{16}$ from place $l_{35}$ splits into two tokens – the same $\alpha_{16}$ which enters place $l_{37}$ without new characteristic and token $\alpha_{21}$ which enters place $l'_{cr}$ where it merges with token $\alpha'_{cr}$. Token $\alpha'_{cr}$ obtains the characteristic *"current value of Ycr"*.

$$Z_{24} = \langle \{l_{37}, l'_{ac}\}, \{l_{38}, l_{39}, l'_{ac}\}, r_{24} \rangle,$$

where

$$r_{24} = \begin{array}{c|ccc} & l_{38} & l_{39} & l'_{ac} \\ \hline l_{37} & W_{37,38} & false & W_{37,ac} \\ l'_{ac} & false & true & true \end{array}$$

and

- $W_{37,ac}$ = "the current request is abandoned (with given probability)";
- $W_{37,38} = \neg W_{37,ac}$.

When the truth value of predicate $W_{37,38}$ is "true" token $\alpha_{16}$ from place $l_{37}$ enters place $l_{38}$ without obtaining new characteristic. When the truth value of $W_{37,ac}$ is "true" token $\alpha_{16}$ enters place $l'_{ac}$ where it merges with token $\alpha'_{ac}$. Token $\alpha'_{ac}$ splits into two tokens – the same $\alpha'_{ac}$ which remains in place $l'_{ac}$ with characteristic *"current value of Yar"* and token $\alpha_{22}$ which enters place $l_{39}$ without obtaining new characteristic.

$$Z_{25} = \langle \{l_{38}, l'_{cc}\}, \{l_{40}, l'_{cc}\}, r_{25} \rangle,$$

where

$$r_{25} = \begin{array}{c|cc} & l_{40} & l'_{cc} \\ \hline l_{38} & true & true \\ l'_{cc} & false & true \end{array}.$$

Token $\alpha_{16}$ from place $l_{38}$ splits into two tokens – the same $\alpha_{16}$ which enters place $l_{40}$ without new characteristic and token $\alpha_{23}$ which enters place $l'_{cc}$ where it merges with token $\alpha'_{cc}$. Token $\alpha'_{cc}$ obtains the characteristic *"current value of Ycc"*.

### 3.4 Generalized net model of the Communication stage

The graphical representation of the GN model of the Communication stage of an overall telecommunication network with queuing is shown in Fig. 6. It consists of 4 transitions and 12 places. The transitions represent functions of base virtual devices as follows:

- $Z_{26}$ represents the function of the Switch device after the base virtual device **cr** in Fig. 2;
- $Z_{27}$ represents the function of Switch device after the **ac** device in Fig. 2;
- $Z_{28}$ represents the function of the **cc** device in Fig. 2;
- $Z_{29}$ represents the function of the Switch device after the **cc** device.

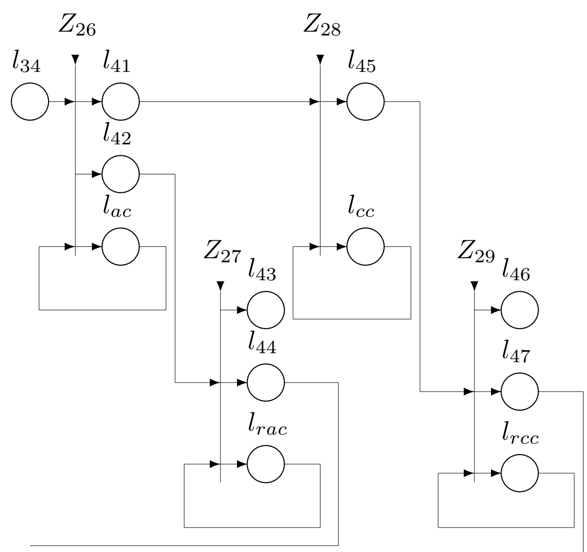


Fig. 6. Generalized net model of the Communication stage of an overall telecommunication system with queuing.

Among the 12 places, 4 places correspond to base virtual devices. These are the places with labels $l_{ac}, l_{rac}, l_{cc}, l_{rcc}$. Tokens $\alpha_{ac}, \alpha_{rac}, \alpha_{cc}$, $\alpha_{rcc}$ stay in the initial time moment in places $l_{ac}, l_{rac}, l_{cc}, l_{rcc}$ respectively. Each of them has initial characteristic *"initial values of* $Y_x, P_x, F_x, T_x, N_x$*"*, where $x$ is the name of the corresponding base virtual device.

Below follows the description of the transitions.

$$Z_{26} = \langle \{l_{34}, l_{ac}\}, \{l_{41}, l_{42}, l_{ac}\}, r_{26} \rangle,$$

where

$$r_{26} = \begin{array}{c|ccc} & l_{41} & l_{42} & l_{ac} \\ \hline l_{34} & W_{34,41} & false & W_{34,ac} \\ l_{ac} & false & true & true \end{array}$$

and

- $W_{34,ac}$ = "the current request is abandoned (with a given probability)";
- $W_{34,41} = \neg W_{34,ac}$.

When the truth value of predicate $W_{34,41}$ is "true" token $\alpha_1$ from place $l_{34}$ enters place $l_{41}$ without obtaining new characteristic. When the truth value of $W_{34,ac}$ is "true" token $\alpha_1$ enters place $l_{ac}$ where it merges with token $\alpha_{ac}$. Token $\alpha_{ac}$ splits into two tokens – the same $\alpha_{ac}$ which remains in place $l_{ac}$ with characteristic *"current value of Yac"* and token $\alpha_{24}$ which enters place $l_{42}$ without obtaining new characteristic.

$$Z_{27} = \langle \{l_{42}, l_{rac}\}, \{l_{43}, l_{44}, l_{rac}\}, r_{27} \rangle,$$

where

$$r_{27} = \begin{array}{c|ccc} & l_{43} & l_{44} & l_{rac} \\ \hline l_{42} & W_{42,43} & false & W_{42,rac} \\ l_{rac} & false & true & true \end{array}$$

and

- $W_{42,43}$ = "the current request is terminated (with a given probability)";
- $W_{42,rac}$ = "the current request becomes repeated request (with a given probability".

When the truth value of predicate $W_{42,43}$ is "true" token $\alpha_{24}$ from place $l_{42}$ enters place $l_{43}$ without obtaining new characteristic. When the truth value of $W_{42,rac}$ is "true" token $\alpha_{24}$ enters place $l_{rac}$ where it merges with token $\alpha_{rac}$. Token $\alpha_{rac}$ splits into two tokens – the same $\alpha_{rac}$ which remains in place $l_{rac}$ with characteristic *"current value of Yrac"* and token $\alpha_{25}$ which enters place $l_{44}$ without obtaining new characteristic.

$$Z_{28} = \langle \{l_{41}, l_{cc}\}, \{l_{45}, l_{cc}\}, r_{28} \rangle,$$

where

$$r_{28} = \begin{array}{c|cc} & l_{45} & l_{cc} \\ \hline l_{41} & true & true \\ l_{cc} & false & true \end{array}.$$

Token $\alpha_1$ from place $l_{41}$ splits into two tokens – the same $\alpha_1$ which enters place $l_{45}$ without new characteristic and token $\alpha_{26}$ which enters place $l_{cc}$ where it merges with token $\alpha_{cc}$. Token $\alpha_{cc}$ obtains the characteristic *"current value of Ycc"*.

$$Z_{29} = \langle \{l_{45}, l_{rcc}\}, \{l_{46}, l_{47}, l_{rcc}\}, r_{29} \rangle,$$

where

$$r_{29} = \begin{array}{c|ccc} & l_{46} & l_{47} & l_{rcc} \\ \hline l_{45} & W_{45,46} & false & W_{45,rcc} \\ l_{rcc} & false & true & true \end{array}$$

and

- $W_{45,46}$ = "the current request is terminated (with given probability)";
- $W_{45,rcc}$ = "the current request becomes repeated request (with given probability".

When the truth value of predicate $W_{45,46}$ is "true" token $\alpha_1$ from place $l_{45}$ enters place $l_{46}$ without obtaining new characteristic. When the truth value of $W_{45,rcc}$ is "true" token $\alpha_1$ enters place $l_{rcc}$ where it merges with token $\alpha_{rcc}$. Token $\alpha_{rcc}$ splits into two tokens – the same $\alpha_{rcc}$ which remains in place $l_{rcc}$ with characteristic *"current value of Yrcc"* and token $\alpha_{27}$ which enters place $l_{47}$ without obtaining new characteristic.

## 4 Analytical modeling of the overall telecommunication system using the GN model

In the GN model described in the previous section the functions of at least 36 important virtual devices are included. Of them 32 are base virtual devices and 4 (**a, b, s, ab**) are comprise (not shown in the graphical representation). They are of interest because the values of their parameters characterize the state of the overall telecommunication system. Every device has five parameters: $P, F, T, Y$ and $N$. Therefore the total number of parameters is 180.

### 4.1 Static and dynamic parameters of the model

In order to construct relatively simple analytical model of the overall telecommunication system, the notions of *system tuple* and *base tuple* have to be introduced.

**Definition 1** *A system tuple is a finite set of distinguishable (by name and/or position) parameters' values, which fulfills simultaneously the three following requirements:*

1. *All parameters (parameters' set), evaluated by the system tuple, correspond to one considered (observed, modeled) system;*
2. *All the values of a system tuple correspondent to one and the same time interval of measurements or considerations;*

3. *The instant of beginning and duration of this time interval are elements of the system tuple set.*

Every subset of a system tuple is called *subtuple*.

There are many obvious dependencies in a system tuple, corresponding to the full parameters' set of the conceptual model. For example, the sum of probabilities of outgoing transitions in every virtual switch devices has value one; in stationary state Little's formula ($Y = FT$) can be applied to every virtual device; we assume that most of the devices have infinite capacity. As a result, there are sets of base parameters (sub-tuples), with the following property: if we know the values of the base parameters, we may calculate the values of all other parameters of the same system tuple. Several different base parameters' sets may exist. After careful analysis and some assumptions (see below) we have chosen a base parameters' set with 46 parameters. The values of these parameters we call *base tuple*. The base tuple is a sub-tuple of a system tuple.

In this paper, we propose a short term classification of the chosen base parameters' set into static and dynamic parameters. For the static parameters we assume that their values do not depend on the state of the system and correspondingly on the intensity of the input flow. They may depend on other factors, e.g. the time of the day; seasons, human temperament, Telecom Administration and so on, but for the observed and modeled time interval we consider them as constants.

- The static parameters are: $M, Nab, Ns, Ted, Pad, Tad, Prad, Pid, Tid, Prid, Tcd, Tbws, Prbws, Pis, Tis, Pris, Pns, Tns, T^{*}cs, Prns, Tbr, Prbr, Par, Tar, Prar, Tcr, Pac, Tac, Prac, Tcc, Prcc$.
- The dynamic parameters, with mutually dependent values are: $Fo$, $Yab, Fa$, $dem.Fa, Tcs, rep.Fa, Pbws, Pbr, ofr.Fws, Tws, Ys, crr.Fs, Fs, Ts$.

### 4.2 Main assumptions

Due to the complexity of the model, in order to obtain relatively simple analytical expressions about the parameters, after a careful analysis the following assumptions are formulated:

A-1. (Closed System Structure) We consider a closed telecommunication system with functional structure shown in Fig. 3 - Fig. 6;

A-2. (Device Capacity) All base virtual devices in the model have unlimited capacity. Comprise devices are limited: **ab**-device contains all the active terminals $Nab \in [2, \infty]$; switching system (**s**) has capacity of $Ns$ calls (every internal switching line may carry only one call); every terminal has capacity of one call, common for both incoming and outgoing calls;

A-3. (A-Terminal Occupation) Every call, from the flow incoming in the telecommunication system ($inc.Fa$), falls only on a free terminal. This terminal becomes a busy A-terminal;

A-4. (Stationarity) The system is in stationary state. This means that in every virtual device in the model (including comprising devices like switching system), the intensity of input flow $F(0,t)$, call holding time $T(0,t)$ and traffic intensity $Y(0,t)$ in the observed interval $(0,t)$ converge to the correspondent finite numbers $F, T$ and $Y$, when $t \to \infty$. In this case we may apply the theorem of Little (1961) and for every device: $Y = FT$;

A-5. (Calls' Capacity) Every call occupies one place in a base virtual device, independently from the other devices (e.g. a call may occupy one internal switching line, if it find free one, independently from the state of the intent B-terminal (busy or free));

A-6. (Environment) The calls in the communication systems' environment don't occupy any telecommunication systems' device and therefore they do not create communication systems' load. (For example, unsuccessful calls, waiting for the next attempt, are in "the head" of the user only. The calls and devices in the environment form the intent and repeated calls flows). Calls leave the environment (and the model) in the instance they enter a Terminator virtual device;

A-7. (Parameters' independence) We consider probabilities for direction of calls to, and holding times in the base virtual devices as independent of each other and from the intensity $Fa = inc.Fa$ of the incoming flow of calls. The values of these parameters are determined by users' behavior and technical characteristics of the communication system. (Obviously, this is not applicable to the devices of type Enter Switch, corresponding to $Pbws$ and $Pbr$.);

A-8. (Randomness) All variables in the analytical model may be random and we are working with their mean values, following the Theorem of Little.

A-9. (B-Terminal Occupation) Probabilities of direction of calls to, and duration of occupation of devices **ar, cr, ac** and **cc** are the same for A and B-calls;

A-10. (Channel Switching) Every call occupies simultaneously places in all the base virtual devices in the telecommunication system (comprised of devices **a** or **b**) it passed through, including the base device where it is in the moment of observation. Every call releases all its occupied places in all base virtual devices of the communication system, in the instant it leaves comprising devices **a** or **b**.

A-11. (Homogeinity of the terminals) All terminals are homogenous, i.e., for every terminal all corresponding characteristics are equal.

A-12. (Direction of the A-calls) Every A-terminal generates all call attempts only towards other terminals, not towards itself.

A-13. (Ordinadirity of the B-flow) The flow directed to the B-terminals ($Fb$) is ordinary. The only exception is when two or more calls reach a free B-terminal simultaneously.

A-14. (Probability of repeated calls blocking) The probability $Pbr$ for finding the B-terminal busy is one and the same for the first and all of the next repeated attempts.

A-15. The probabilities $Pad, Pid, Pbws, Pis, Pns, Pbr, Par, Pac$ preserve their values during the repeated call attempts.

A-16. The probabilities of entering of repeated call attempts due to abandoned dialing $Prad$, interrupted dialing $Prid$, interrupted switching $Pris$, not available switching $Prns$, abandoned ringing $Prar$, abandoned communication $Prac$, blocked waiting for switch ($Prbws$) and blocked ringing ($Prbr$) which characterize the users' behavior also preserve their values.

A-17. Parameters characterizing the users' behavior are the mean service time of the separate devices: $Ted$, $Tad$, $Tid$, $Tcd$, $Tbws$, $Tis$, $Tns$, $T^*cs$, $Tbr$, $Tar$, $Tcr$, $Tac$ and $Tcc$. They remain the same for the repeated call attempts.

### 4.3 Equation for the traffic intensity of the called terminals

We shall illustrate how equations for the dynamic parameters of the system can be obtained using the GN model and the assumptions. As an example, the traffic intensity of the called terminals is chosen.

**Theorem 1.** *The traffic intensity of the B-terminals ($Yb$) can be determined from the equation*

$$Yb = Fb\,Tb\,, \tag{1}$$

*where $Fb$ is the flow intensity of the B-terminals and $Tb$ is the mean holding time of calls in a B-terminal, and:*

$$Fb = Fa(1 - Pad)(1 - Pid)(1 - Pbws)(1 - Pis)(1 - Pns)(1 - Pbr)\,, \tag{2}$$

$$Tb = ParTar + (1 - Par)[Tcr + PacTac + (1 - Pac)Tcc]\,. \tag{3}$$

Proof: Equation (1) is the Little's formula for device **b** in stationary state (A-4). Equation (2) expresses the fact that the A-calls have to avoid the six modeled losses before occupying the intent B-terminals, with mean intensity of calls $Fb$. This is seen from the graphical representation of the GN models of the four stages. Equation (3) is direct corollary from the GN model in which the B-terminals are represented by transitions $Z_{35}, Z_{36}, Z_{37}$ and $Z_{38}$, the closed system structure (A-1), calls' capacity (A-5), excluding calls in the environment (A-6) parameters independance (A-7), randomness (A-8) and B-terminal occupation assumption (A-9).

We may derive the expression (3) for the B-terminals holding time ($Tb$) from the following considerations. From the graphical representation of the GN model, parameters independence (A-7) and channel switching (A-10), it follows that $Yb$ is a sum of the traffics of the base blocks, comprised in it. The assumption for B-terminal occupation (A-9), implies that $Yar$, $Ycr$, $Yac$ and $Ycc$ are the same traffic intensities for A and B-terminals, so:

$$Yb = Yar + Ycr + Yac + Ycc. \tag{4}$$

On the other hand, we may express the traffic intensities using the Little's formula and presenting every flow intensity in the base devices as a function of $Fb$:

$$Yar = FarTar = FbParTar\,; \quad (5)$$

$$Ycr = FcrTcr = Fb(1 - Par)Tcr\,; \quad (6)$$

$$Yac = FacTac = Fb(1 - Par)PacTac\,; \quad (7)$$

$$Ycc = FccTcc = Fb(1 - Par)(1 - Pac)Tcc\,. \quad (8)$$

After replacing (5), (6), (7) and (8) in (4), taking in consideration A-9 and using (1) we obtain (3). □

Equations for the rest of the dynamic parameters can be obtained in a similar way using the approach from [8].

## 5 Conclusions

The GN model of overall telecommunication system with queuing described in the present paper has the following important features:

- The virtual devices are represented only by three objects: transitions, arcs and places of the GNs.
- The repeated flows of requests are represented graphically by the arcs of the GN and the path of the requests can be easily tracked.
- The graphical representation of the GN can be used for the derivation of equations about the parameters of the telecommunication system.

Some problems related to the use of GNs as a tool for conceptual modeling of telecommunication systems which arise are:

- The functions of the virtual devices are not evident from the graphical representation of the GN. They are described in the transitions' components. This requires knowledge of the theory of the GNs in order to understand the model.
- The problem for representation of comprise virtual devices in GNs should be studied. Different representations are possible: using additional transitions; additional places; including the parameters of the comprise devices as characteristics of specific tokens, etc.
- The huge number of virtual devices requires the graphical representation of the GN model to be divided into smaller parts.

Some of the problems above can be solved using extensions of the ordinary GNs.

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